Radiation
[Classical Theory]

- Semester 6
- Physics Honours
- Hooghly Women's College
- Course title: CC-14: Statistical Mechanics

Classical Theory of Radiation: Properties of Thermal Radiation. Blackbody Radiation.Pure temperature dependence. Kirchhoff's law. Stefan-Boltzmann law: Thermodynamic proof. Radiation Pressure.Wien's Displacement law. Wien's Distribution Law. Saha's Ionization Formula. Rayleigh-Jean's Law.Ultraviolet Catastrophe. (9 Lectures)

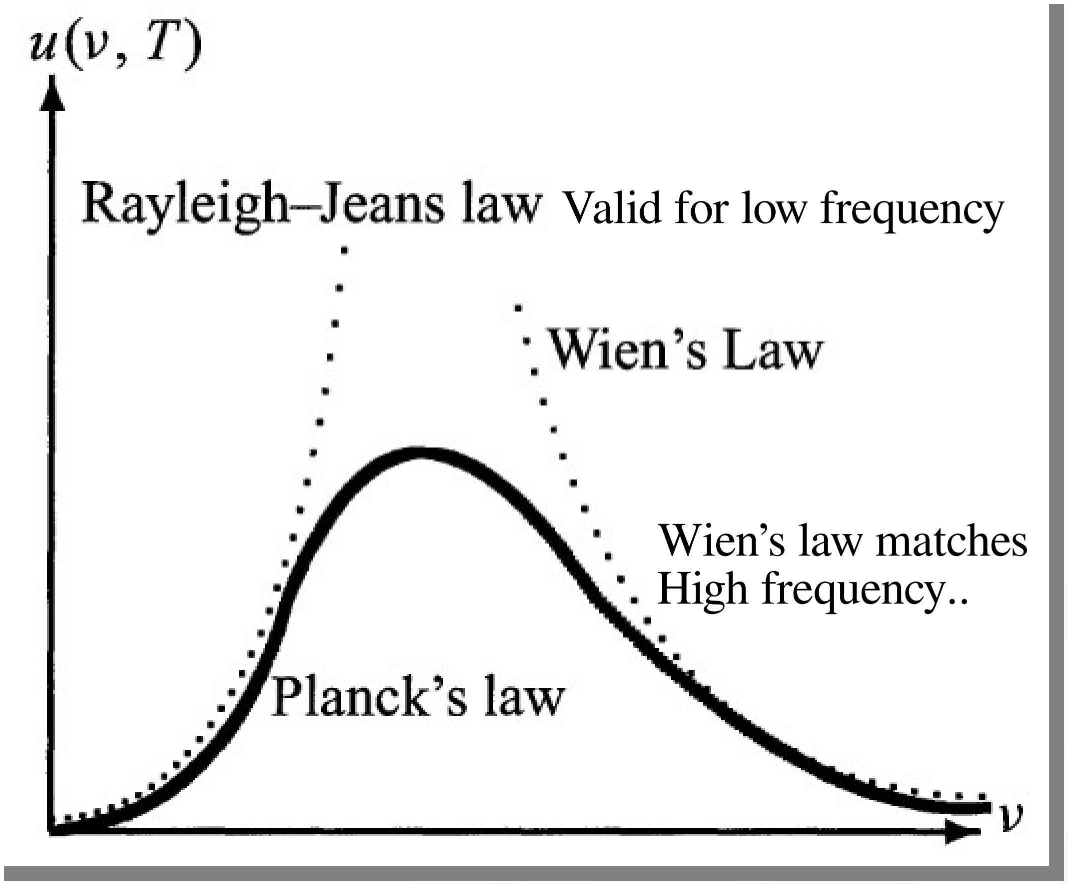
Properties of Thermal Radiation

- Thermodynamics laws are applicable to Black body. (What is Black Body?)
- Inside an enclosure there is a complete absorber or blackbody, and the walls of enclosure is at fixed temperature T
- Experimental result:
  - Radiant energy per unit area is function of temperature T
  - Does not depend on volume or shape of enclosure
  - The thermal radiation emitted by glowing solid objects consists of a continuous distribution of frequencies ranging from infrared to ultraviolet.
  - The energy density shows a pronounced maximum at a given frequency, which increases with temperature; that is, the peak of the radiation spectrum occurs at a frequency that is proportional to the temperature

Blackbody Radiation

Pure temperature dependence

This solid line was experimental curve, which did not let the theoretical Physicists sleep in peace. Every one tried his/her luck with this problem..



Wien's law fails at
low frequency..

Kirchhoff’s Law

- A good emitter of radiation is also a good absorber of radiation, and vice versa.

– qualitative statement of “Kirchoff’s Law”

From the book by Reif:

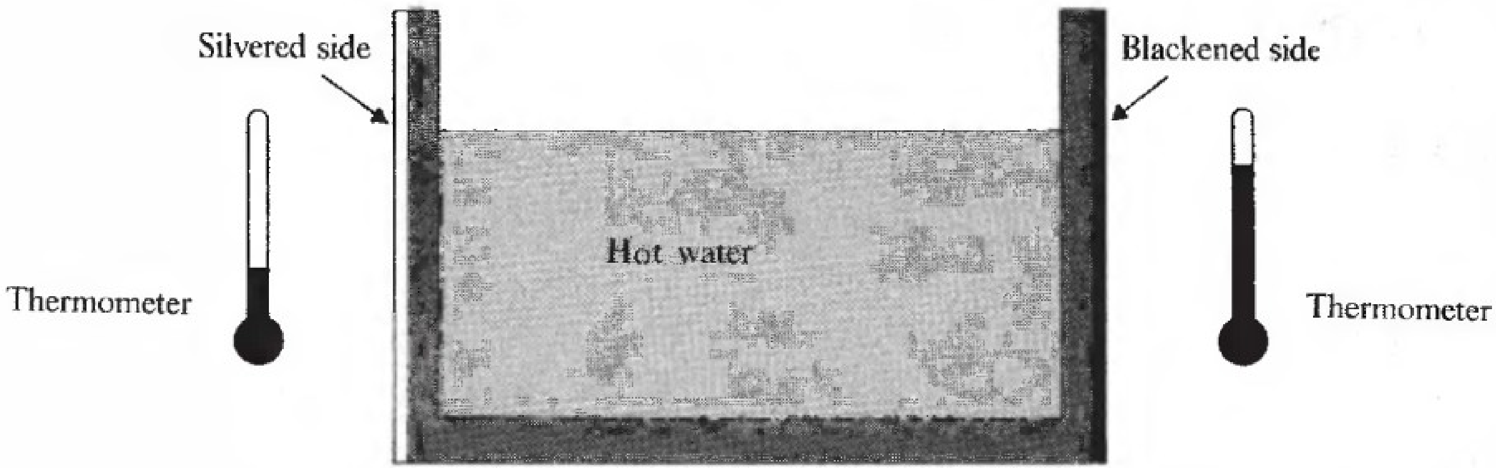


***Fig. 9·15·3** A classical experiment illustrating Kirchhoff’s law. The container is filled with hot water. Its left side is silvered on the outside so that it is a poor absorber; its right side is blackened so that it is a good absorber. Since the left side is then a poorer emitter of radiation than the right side, the thermometer on the left is found to indicate a lower temperature than the one on the right.*

Kirchhoff's Law

- A good emitter of radiation is also a good absorber of radiation, and vice versa.

– qualitative statement of "Kirchoff's Law"

From the book by Reif:

Remark Kirchhoff's law is a reasonable result by virtue of the following microscopic considerations. Focus attention on a pair of energy levels of the body; transitions between these give rise to emission or absorption of radiation at some frequency ω. If transitions between these levels are readily produced (i.e., if the transition probability is large), then the electric field in the incident radiation can readily induce absorption in transitions from the lower to the upper level; but then the thermal agitation can also readily induce emission in transitions from the upper to the lower level.

Stefan-Boltzmann law: Thermodynamic proof

Derivation of Stefan's Law

$$u(T) = \frac{U(T)}{V}; \qquad U = TdS - PdV; \qquad (\frac{\partial S}{\partial V})_T = (\frac{\partial P}{\partial T})_V \tag{35}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T(\frac{\partial S}{\partial V})_T - P = T\left(\frac{\partial P}{\partial T}\right)_V - P \tag{36}$$

$$U(T) = u(T)V; \qquad P = \frac{1}{3}u(T) \tag{37}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = u(T); \qquad \left(\frac{\partial P}{\partial T}\right)_V = \frac{1}{3}\left(\frac{\partial u}{\partial T}\right)_V \tag{38}$$

$$u = \frac{T}{3}\frac{du}{dT} - \frac{u}{3}; \qquad \frac{du}{u} = 4\frac{dT}{T} \tag{39}$$

$$u = \sigma T^4 \tag{40}$$

This is Stefan's constant σ

Stefan-Boltzmann law: Thermodynamic proof

$$\text{power per unit area} = \frac{2\pi^5}{15}\frac{(kT)^4}{h^3c^2} = \sigma T^4,$$

σ is known as the **Stefan-Boltzmann constant,**

$$\sigma = \frac{2\pi^5 k^4}{15h^3c^2} = 5.67 \times 10^{-8}\ \frac{\text{W}}{\text{m}^2\,\text{K}^4}.$$

Radiation Pressure

- Derive yourself..
- Prove that $P = u/3$

Wien’s energy distribution law

$$u(\nu, T) = A\nu^3 e^{-\beta\nu/T},$$

Wien took Stefan’s law and using thermodynamic relations, showed above relation.

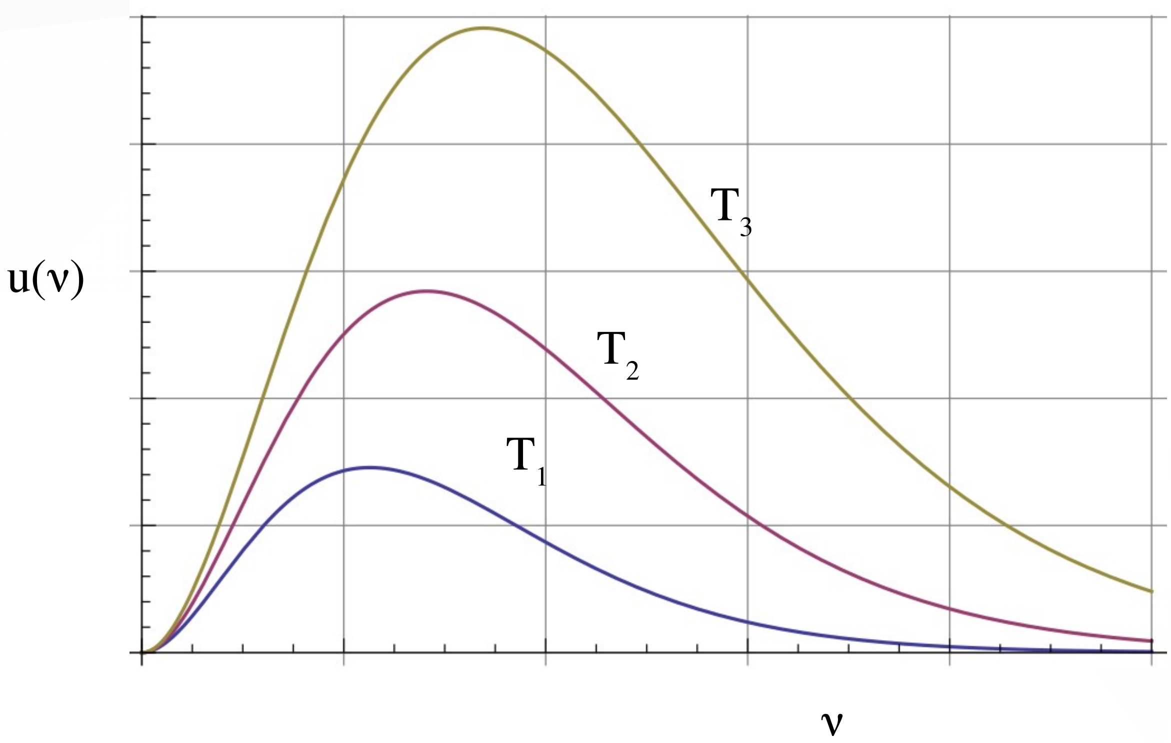
where A and β are two parameters that can be adjusted to fit the experimental data.

u(ν,T) has the dimensions of an energy per unit volume per unit frequency. Although Wien's formula fits the high-frequency data remarkably well, it fails badly at low frequencies.

Wien's Displacement law

Here ν_{max} proportional to temperature

Defined: ν_{max} is where energy density is max at that frequency



Prove this..

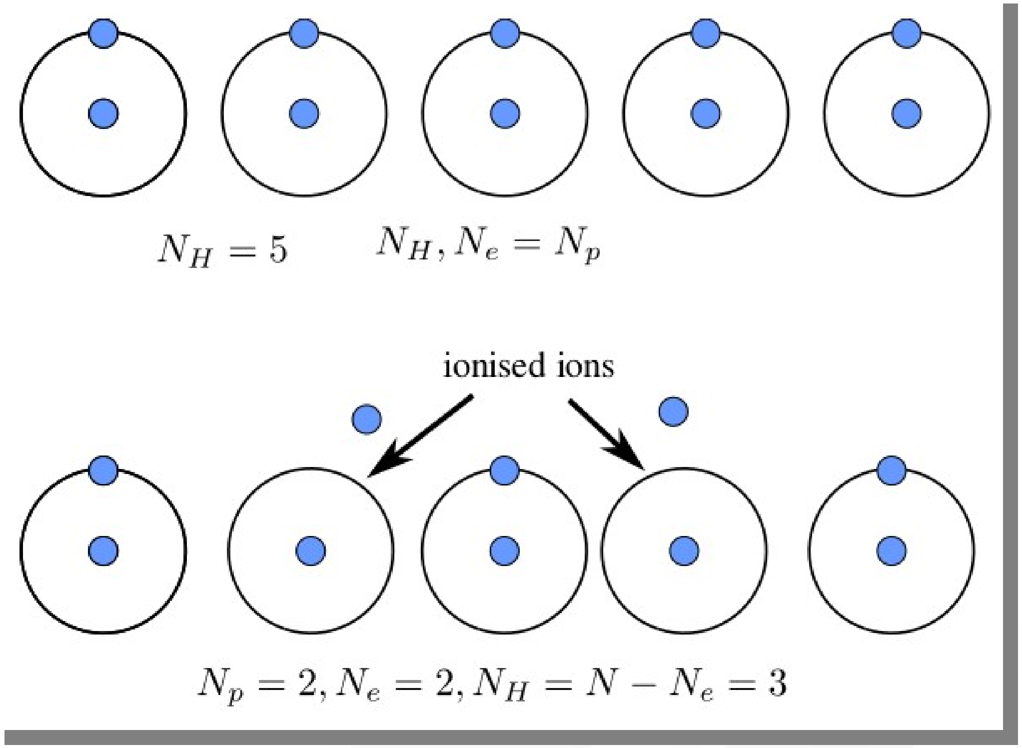
$$\nu_{max} = bT$$

This is experimental observation aslo

Home Work: Take derivative of energy density with respect to frequency and prove Wien's displacement law

Saha's Ionization Formula

- Meghnad Saha in 1920
- Astrophysical context:



The Saha equation gives a relationship between free particles and those bound in atoms. When en electron free itself from the coulomb potential, its gained Ionization energy

$$I = -E = -\left(-\frac{13.6\ Z^2}{n^2}\right) = \frac{13.6\ Z^2}{n^2} \quad (1)$$

Saha's Ionization Formula

Let us consider a gas of N number of atoms in which there are N_e number of electrons are unbound and N_p number of protoms are also there ($N_e = N_p$) and rest (N-N_e) are simple neutral atoms.

The partition function of each class of particles namely electrons, protons and H-atoms are

$$Z_e = \sum_n e^{-\beta E_e} \tag{2}$$

$$Z_p = \sum_n e^{-\beta E_p} \tag{3}$$

$$Z_H = \sum_n e^{-\beta E_H} \tag{4}$$

The probability function P (not entropy or simple probability, but this is function of probability function)

$$P(N_e, N) = \frac{Z_e^{N_e}}{N_e!}\frac{Z_p^{N_p}}{N_p!}\frac{Z_e^{N_H}}{N_H!} \tag{5}$$

Let us now calulate what is Z in general. index i will be replaced by e or p.

$$Z_i = g_i \int e^{-\beta p^2/2m} \frac{d^3x d^3p}{h^3} = g_i \frac{V}{h^3}(2\pi m k_B T)^{3/2} \tag{6}$$

Now $g_e = g_p = 2$ and $g_H = 4$ these are statistical weight factor. These are 2 for fermions.

Saha's Ionization Formula

$$Z_e = \frac{2V}{h^3}(2\pi m_e k_B T)^{3/2}$$
$$Z_p = \frac{2V}{h^3}(2\pi m_p k_B T)^{3/2}$$

$$Z_H = g_H \int e^{-\beta(\frac{p^2}{2m_H} - I)} \frac{d^3x d^3p}{h^3} = \frac{4V}{h^3}(2\pi m k_B T)^{3/2}\ e^{\beta I}$$

$$P(N_e, N) = \frac{Z_e^{N_e}}{N_e!} \frac{Z_p^{N_p}}{N_p!} \frac{Z_e^{N_H}}{N_H!}$$

$$lnP = N_e ln Z_e + N_p ln Z_p + N_H ln Z_H - (N_e ln N_e - N_e + N_p ln N_p - N_p + N_H ln N_H - N_H)(10)$$

Saha's Ionization Formula

$$\frac{dlnP}{dN_e} = lnZ_e + lnZ_p - lnZ_H - 2lnN_e + ln(N - N_e) = 0$$

$$\frac{N_e^2}{N - N_e} = \frac{V}{h^3}(2\pi m_e k_B T)^{3/2}\ e^{-\beta I}$$

By setting $n_e = \frac{N_e}{V}$, $n = N/V$ and writing $x = \frac{n_e}{n}$ we get the Saha ionization equation

$$\frac{x^2}{1 - x} = \frac{1}{nh^3}(2\pi m_e k_B T)^{3/2}\ e^{-\beta I}$$

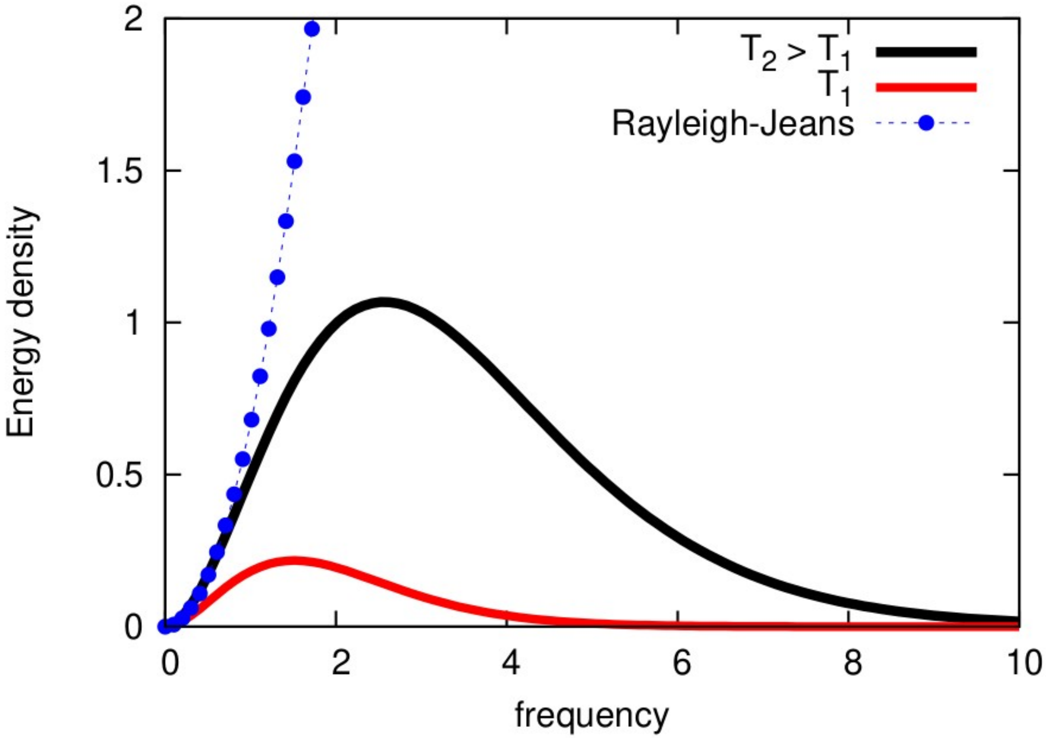
Can be used to find temperature of the Sun or any star in universe.
Can be used to find ionization energy of elements.

Rayleigh-Jean's Law

Wien's distribution law matched the high frequency part of Blackbody Radiation formula.

$$N(\nu) = \frac{8\pi \nu^2}{c^3},$$

- Rayleigh tried his luck in 1900.
- He fitted the curve for low frequency.



$$u(\nu, T) = N(\nu)\langle E\rangle = \frac{8\pi \nu^2}{c^3}\langle E\rangle,$$

Classical here!

$$u(\nu, T) = \frac{8\pi \nu^2}{c^3} kT.$$

Ultraviolet Catastrophe

- Failure of Rayleigh-Jeans formula: at high frequency.
- Experiment says: At high frequency, energy density should be low.
- Rayleigh-Jean’s equation says: Energy increases with frequency.
- This is called **Ultraviolet Catastrophe**

The solution from this problem was given by Quantum mechanics. Namely $\langle E \rangle$ is not k_B T. but we have to take radiation as quantized energy as oscillators..

Conclusions

- Need for a new theory
- Birth of Quantum Mechanics
- Backed by solid mathematics background
- S N Bose, C V Raman, (all have Kolkata Connection)
- Dirac, Schrodinger, Heisenberg, many others..